DOI: 10.15669/pnst.4.548

Progress in Nuclear Science and Technology
Volume 4 (2014) pp. 548-552

ARTICLE

Parameter search of geometric-progression formula for gamma-ray isotropic point source buildup factors up to depths of 100 mfp, including contribution of secondary radiations

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The form of $K(E,X)$ in the Geometrical Progression (G-P) fitting function for 28 single-material gamma-ray buildup factors was improved to accurately approximate for deep penetration up to 100 mean free paths (mfp). The reference data were generated with Invariant Embedding (IE method) code treated Klein-Nishina scattering cross section accurately, included all secondary radiations of fluorescence and bremsstrahlung, and covered the energy range 0.015-15 MeV and up to 100 mfp. The search procedure of fitting parameters to gamma-ray buildup factors was revised from the least squares fit (LSF) to the minimization of maximum fractional deviation (MMD). Revised $K(E,X)$ has 7 parameters. Here two values of X_k and X_m parameters were fixed, and the remainder parameters were determined using the (MMD). For an energy not listed in the reference, the values of G-P fitting parameters were determined using the interpolation method of Akima. By the same method the values of G-P fitting parameters for equivalent atomic number of a compound or a mixture were determined. The maximum errors of the present fitting parameters within 40 mfp were almost the same values to the ANSI/ANS-1991 or below. Those above 40 mfp were almost the same values within 40 mfp, or a little large.

Keywords: gamma-ray buildup factors; Geometrical-Progression formula

1. Introduction

Analysis of the attenuation of gamma rays from an extended source of radiation, as a preliminary approximation in actual shielding design, is usually estimated by multiplying the uncollided point kernel by the buildup factor represented by the empirical formula like the fitting function of the geometric-progression (G-P) method [1-3].

The empirical formula eliminates the need to interpolate with respect to attenuation distance or to both source energy and distance. The G-P fitting formula which can accurately reproduce the buildup data up to depths of 40 mean free paths (mfp) within a few percent was adopted by American National Standard ANSI/ANS-6.4.3-1991 [2] as the best available form for the fitting function.

The reference data were generated with the Invariant Embedding (IE) method code by A. Shimizu et al. [4] as a function of distance up to 100 mfp for the photon energy from 0.015 to15MeV, where for that from 0.03 to 15MeV for Lead, Bismuth and Uranium.

The form of $K(E,X)$ was improved to accurately approximate for deep penetration up to 100 mfp.

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Previous G-P parameters were determined by using the least squares fit (LSF) [1,2]. Then, there were some cases that the values of interpolated G-P parameters gave the value of incorrect buildup factor. It happens in the case that the value of X_k parameter is small.

The present G-P parameters were determined by using a minimizing procedure of the maximum fractional deviation (MMD) fit. Shultis and Faw compared the (MMD) and the least squares fit (LSF) to logarithmic data criteria for their skyshine dose data [5]. They found that the MMD criterion produced smaller maximum deviations between the fits and the reference values though both fitting criteria produced the same range of average deviation.

Therefore, on the present fitting process the values of the parameters of G-P formula were determined as the fixed values of X_k and X_m fitting parameters: i.e. X_k=15 mfp and X_m=40 mfp from Beryllium to Copper, and X_k=25 mfp and X_m=70 mfp from Ruthenium to Uranium. The remainder parameters of *b*, *c*, *a*, *d* and η were determined using the (MMD).

For an energy not listed in the standard, the values of G-P fitting parameters were determined using Akima's interpolation method [6].

For high-Z materials, the buildup factor near shell edges of photoelectric absorption can become very large due to the non-continuous nature of the cross section. This can be observed in the values for Rubidium and elements of higher Z. Hereafter, materials including the contribution of K-X ray are represented as high-Z. It should be kept in mind that fluctuations in energy of the attenuation factor [$B(E,x)e^{-\mu x}$], given as a function penetration depth in centimeters, are not nearly as great as that of buildup factor. Thus, for energies just above the K edge, interpolation in the attenuation factor is easier than in the buildup factor. The energy interpolation for high-Z is limited to above 0.2MeV.

The maximum errors of the present fitting parameters within 40 mfp are almost the same values or below to the ANSI/ANS-6.4.3. Those above 40 mfp are almost the same values or a little large within 40 mfp. The maximum deviation error of results with G-P fitting parameters from reference data was within 50%.

2. Geometric progression fitting function

A buildup factor function of distance from the source is represented in the form [1].

$$B(E,X) = 1 + (b-1)\frac{K^X - 1}{K - 1} \quad \text{for } K \neq 1 \quad \text{and}$$
$$B(E,X) = 1 + (b-1)X \quad \text{for } K = 1. \qquad (1)$$

Equation (1) is the basic approximating formula of the G-P method for buildup factors. Where X is the distance from the source in mfp. When photons emitted from an initial mono-energetic source arrive at 1 mfp from the source, the un-collided and scattered photons that comprise the basic shape of the energy flux, reflect the characteristics of the medium and the source energy. Hence, the value of the buildup factor at 1 mfp is determined from an integration over this spectrum and represents the basic parameter b for a specific material and energy. The variation of parameter K together with penetration represents the gamma-ray dose multiplication and change in the shape of spectrum from that at 1 mfp, which determine the value of b [1].

A factor $(K^X -1)/(K-1)$ is the G-P formula, which is capable of expressing from as large a number as 10^{34} to as small as 10^0 at the distance X=100 mfp, depending on the value of K.

3. Behavior of K parameter

Four typical examples of Eq.(1), the dependence of log K on log X, are shown for E_0 = 0.1, 1, 4 and 10 MeV with water in Fig.1. The values of K from Eq.(1) are plotted with □, ■, ○ and ●, respectively. The reference data are given at a discrete distance from the specified energy source. Therefore, if the values of K are represented with a continuous formula in regards to distance for a specified energy, it is very straight forward to obtain a value for the buildup factor at an arbitrary distance.

It can be observed from Fig.1 that the parameter K has four characteristic behaviors.

1. A graph of the function log K versus log X appears in a straight line with a slope "a" up to around 15 mfp, where $0 < X \leqq X_k$ mfp.
2. The value of log K deviates from linearity beyond around 15 mfp, and describes a gentle curve up to about 40 mfp, where $X_k < X \leqq X_m$ mfp.
3. The values of parameter log K progress along a straight line beyond about 40 mfp, where $X_m < X \leqq 100$ mfp.
4. A value of parameter K corresponding to X can be greater than unity, less than unity, or almost unity, depending on the source energy.

Cases of downward trend with increasing penetration can be observed for 0.1 and 1 MeV in Fig.1. The energy range of dominant photon intensity progresses from a higher energy to a lower one. The energy range of the dominant contribution shifts to the peak range by the multiple scattering of photons at ~40 keV with increased penetration, where the ratio of photoelectric effect is ~20%. The values of log K for E_0=4 MeV are almost independent of log X. In the case of the upward trend for E_0=10 MeV, the energy range of dominant intensity corresponds to movement from a lower to a higher range. The buildup factor data of ANSI/ANS-6.4.3 were only up to 40 mfp. For distances up to 40 mfp, the behavior of K corresponds to the behavioral characteristics 1 and 2 above.

Characteristic 4 corresponds to the difference in dose-weighted spectra intensity at X_i and X_{i-1}. The difference increases where K >1, decreases where K<1 and remains the same for K=1 [1]. The spectrum transforms gradually slower from approximately X=X_k. Characteristic 2 is represented by the second term on the right side in the following formula.

$$K(E,X) = cX^a + d\,\frac{\tanh(X/X_k - 2) - \tanh(-2)}{1 - \tanh(-2)} \quad \text{for } 0 \leqq X \leqq X_m, \qquad (2)$$

Equation (2) represents the dependence of K on X, and is the fitting formula of the K parameter in Eq.(1), for $0 \leqq X \leqq X_m$. At the start of G-P formula the second term of the right side of Eq.(2) was $d(X-X_k)$ [7]. In order to be rounding off the cross of two straight lines, $d(X-X_k)$ was replaced with the term of tangent-hyperboric. Here a, c, d, X_k and X_m are parameters that depend on the attenuating medium and source energy E.

Furthermore, the values of log K can be observed to be linear at a range between X_m mfp and 100 mfp in **Figure 1**.

In order to improve such behavior, the following formula above X_m is proposed.

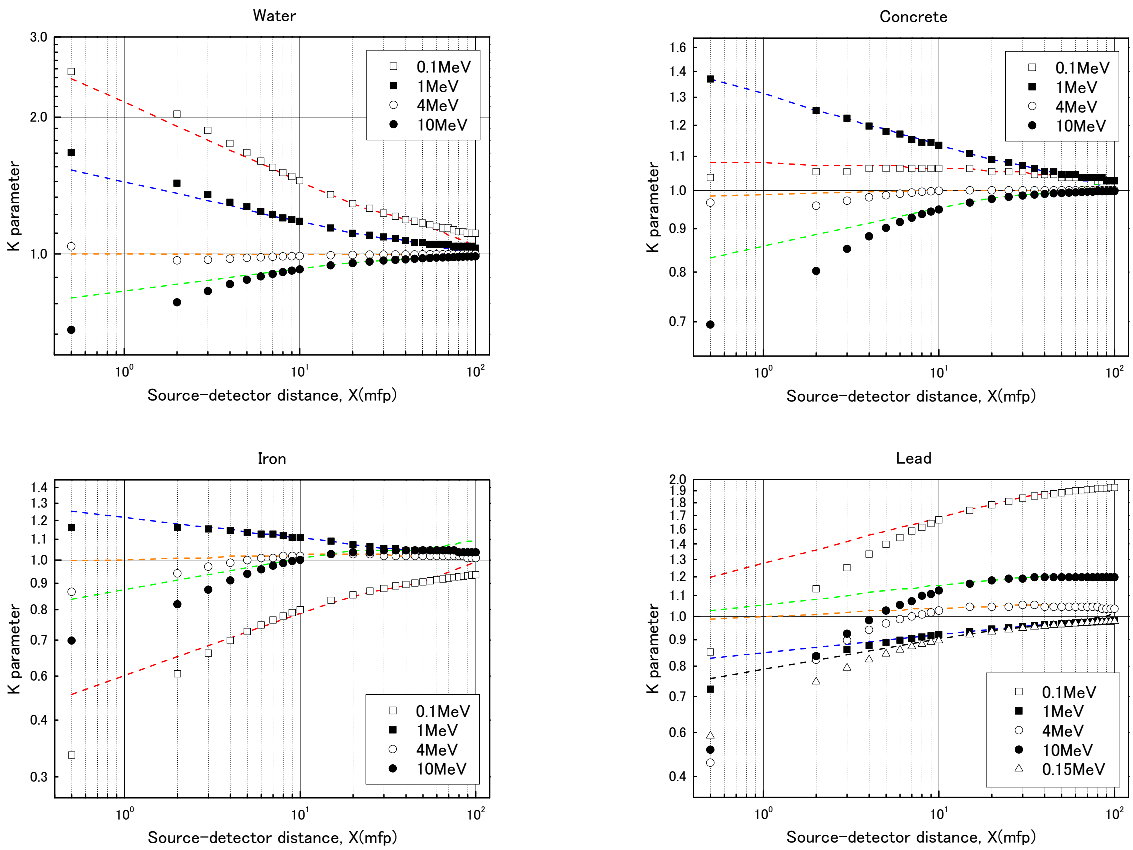

Figure 1. Dependence of log *K* on log *X* for exposure buildup factor, point isotropic sources in infinite media of water, iron, concrete and lead.

$$K(E,X) = 1 + (K_{m-1} - 1)\left[\frac{K_m - 1}{K_{m-1} - 1}\right]^{\xi(X)},$$

where, $$\xi(X) = \frac{(X/X_{m-1})^{\eta} - 1}{(X_m/X_{m-1})^{0.1} - 1}$$

for $X_m < X$. (3)

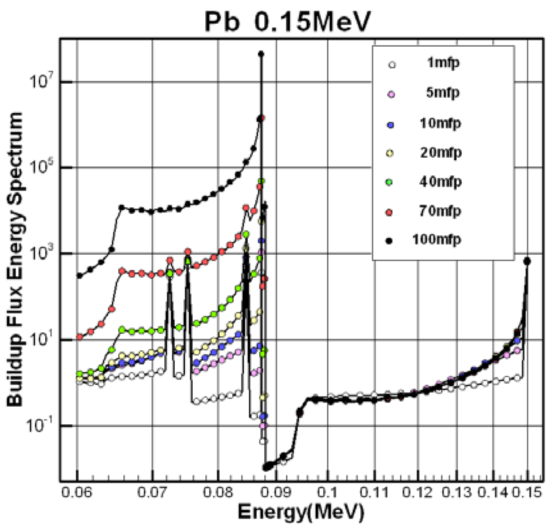

Figure 2. The energy spectra for 0.15 MeV source for various lead thickness base on IE method calculations.

Figure 2 describes behavior of energy spectra of lead at point source 0.15 MeV. The maximum contribution of energy spectra changes remarkably by those of K-X rays between 0.5 to 25 mfp, becomes smaller between 25 to 70 mfp, and do not observe between 70 to 100 mfp. The values of X_k and X_m are fixed 25 mfp and 70 mfp for Ru - U.

The values of X_k and X_m are fixed 15 mfp and 40 mfp for Be - Cu, by the contribution of single scattering gamma ray to energy spectra.

There were some values of *K* obtained by Eq.(1) that deviated from the fitting line near the source. However, these deviations were not critical to the value of buildup factor.

4. Procedure of fitting G-P parameter

The previous G-P parameter set of ANSI/ANS-6.4.3 was determined with the LSF. The present values of the G-P fitting parameters for point source exposure concrete buildup factors up to 100 mfp have been determined using the MMD criteria [3], where X_k and X_m are fixed 15 and 40 mfp. The behavior of G-P parameters of *b*, *c*, *a*, *d* and η for exposure concrete buildup factors are described **Figure 3**. The deviation of η from 0.1 at source energies 0.08 and 4 MeV are seen. This is depending to relieve the sign of *a* and *d* around 0.08 and 4 MeV. Each G-P parameter draws a smooth curve for source energy except above two energies.

There are almost no difference between (LSF) and (MMD) fit to the data of ANSI/ANS-6.4.3 and Atomic Energy Society of Japan (AESJ)-2012 for water, iron and concrete exposure buildup factors by G-P parameters of the range up to 40 mfp. The values of (MMD) up to 100 mfp are almost same to those up to 40 mfp. It is an effect of the secondary radiation of bremsstrahrung to reference data that the values of maximum deviation of G-P fit for concrete and iron at 15MeV in reference data are larger than those of ANSI/ANS-6.4.3.

5. Interpolation of parameters of the G-P formula in E

The interpolation for each parameter with respect to log *E* can be executed by Akima's method [6]. This is a type of spline interpolation method which can treat the data at a singular point. It has been reported from test results for uni-and bi-variable interpolation that the Akima's method is the most effective in every respect [8].

From the sets we selected one where the value of the buildup factor of the G-P parameters interpolated with Akima's method at a specified energy, reproduced accurately as reference data. Reference data of intermediate energy were produced by Akima's interpolation of the original reference data. When the value of the buildup factor calculated using interpolated G-P parameters in log *E* greatly deviated from the

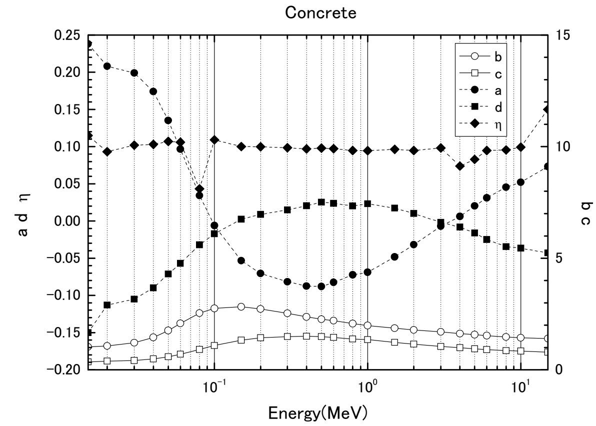

Figure 3. The behavior of G-P parameters on log *X* for exposure buildup factor, point isotropic sources in infinite concrete.

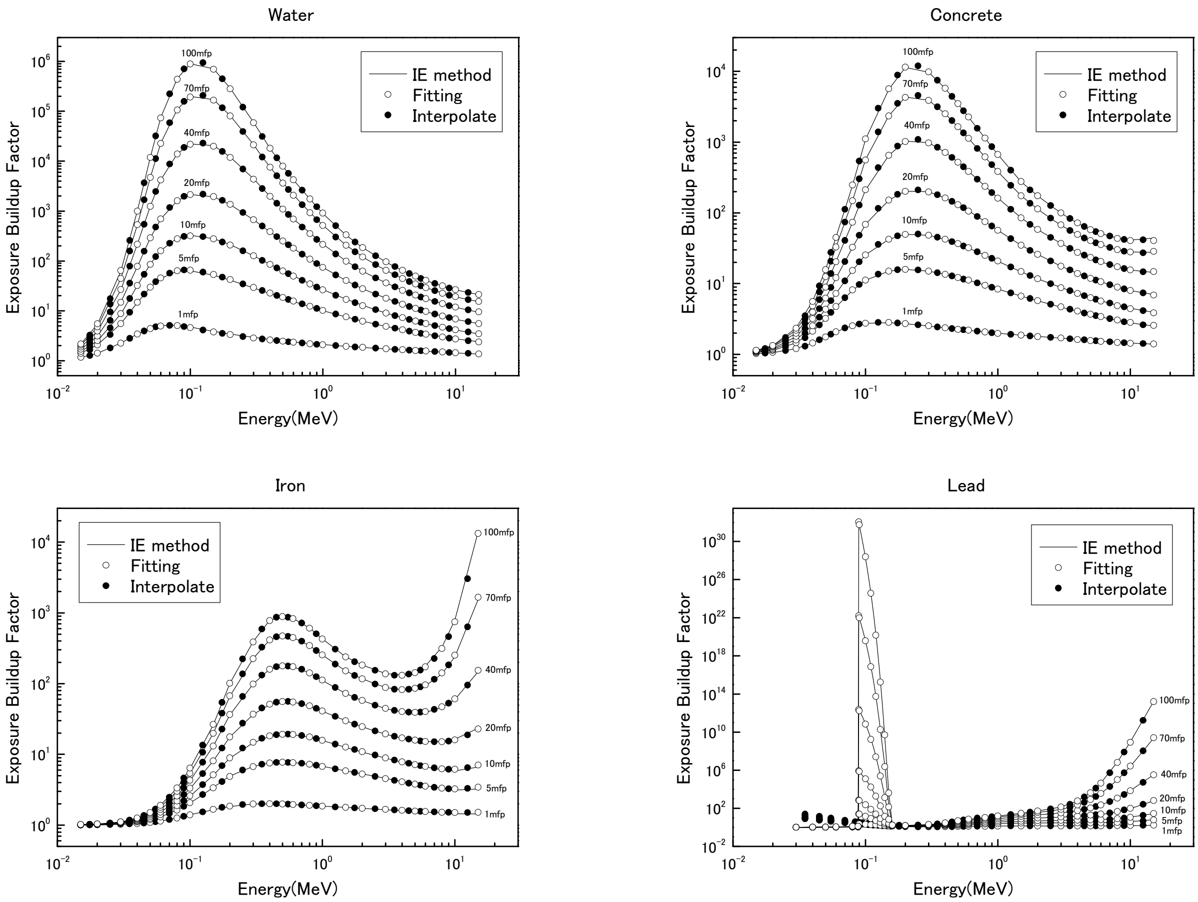

Figure 4. Comparison of the exposure buildup factor in infinite thick water, iron, concrete and lead estimated by the interpolation of seven parameters with the present G-P formula and those from the interpolation of the reference data.

reference data, the third and forth steps were repeated [1, 3].

In **Figure 4** the reference exposure buildup factors of water, iron, concrete and lead media are plotted by a solid line, the values calculated from the G-P parameters by an open circle (○), and those calculated from the interpolated G-P parameters by a filled circle (●). It was ascertained that the G-P fitting parameters reproduced accurately the value of the buildup factor at *X* (mfp) and for *E*.

6. Interpolation of parameters of the G-P formula in atomic number

The estimated values of buildup factor for water, air and concrete of exposure buildup factors are obtained using interpolated G-P parameters for the same equivalent atomic number with Ref.9 by Akima's method. The values of buildup factors for mixtures are accurately reproduced within the maximum deviation 50% reference data.

7. Conclusion

The form of *K*(*E*,*X*) in the G-P formula was improved to accurately approximate for deep penetration up to 100 mfp. The parameter *K* represents appropriately the characteristics of dose variation due to the change in spectrum together with penetration up to 100 mfp, by Eqs.(1), (2) and (3). The improved G-P formula gives an enhanced fit for all the values of buildup factors calculated by the IE method up to 100 mfp within about 15%. Furthermore, the validity of using the G-P parameters to interpolate the buildup factor in *X* (mfp) up to 100 mfp and of using the G-P parameters interpolated with respect to log *E* for the buildup factor at an arbitrary energy between the calculated reference data were ascertained. Consequently, discrete buildup data fitted by the G-P formula was converted to continuous data with regard to both energy and distance.

Furthermore, G-P fitting parameters will be able to obtain a buildup factor for a mixture.

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